SEISMIC RESPONSE REDUCTION FACTOR, OVER-STRENGTH FACTOR AND DUCTILITY OF RC BUILDINGS USING NONLINEAR PUSHOVER ANALYSIS

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ABSTRACT

Response reduction factor R, which is used to reduce the elastic inertia force caused by earthquakes, is one of the most crucial design factors for earthquake-resistant structures. The strength and ductility capacities corresponding to any particular R value play a significant role in the nonlinear response of moment-resisting RC structures. The over-strength, ductility, redundancy, and damping are some of the structural parameters that affect the response reduction factor. This study focuses on assessing the R values for special moment-resisting RC frames and the variation of over-strength and ductility factors in RC buildings with different seismic zones and the number of stories. In order to estimate the R factor, RC buildings with special moment resisting frames are subjected to nonlinear static pushover analysis. For this study, the building model of G+3, G+6, G+9, G+12 stories for Dhaka, Chittagong, Sylhet which are located in seismic zone 2 (zone factor=0.20), zone 3 (zone factor=0.28) and zone 4 (zone factor=0.36) of Bangladesh respectively have been modelled and analyzed by ETABS (2017). After doing the pushover analysis of the buildings, the pushover capacity curves are bi-linearized as per FEMA-356 (2000) guideline. The values of over-strength factor and ductility factor are estimated by using the bi-linearized pushover curve. Using the relationship between ductility and over-strength factor, the response reduction factor R of RC buildings is assessed. It is observed that the over-strength factor changes according to seismic zones and the natural time of the building frames. The over-strength factor decreases with the increase of time period and zone factor respectively. As the natural time period of the structure increases, the ductility of the building increases. It is also noted that the ductility increases as the zone factor increases for all four types of buildings. The seismic zones affect the overall performance of the structure. As the seismic zone increases, the overall seismic response reduction factor, which is dependent on ductility and over-strength components, drops. Increase in number of stories gives different values of time period and R factor, it indicates that R factor does not have a constant value. There is a relationship between R factor and time period of the building. The result also shows that the estimated value of R is almost identical to the R value obtained from Bangladesh National Building Code (BNBC) 2020 since the values are 1.25% to 11.25% higher than code value of R. As all values of R are higher than BNBC (2020) specified value for SMRF building of 8, it indicates that the structures will provide better performance in terms of seismic resistance than the requirements set by the code. So, the values of the over-strength, ductility, and response reduction factors are found to be strongly influenced by the seismic zones and time periods of the structures.

Keywords: *Response reduction factor, pushover analysis, ductility factor, over-strength factor, SMRF.*

1. INTRODUCTION

Strength and ductility properties must be carefully considered when designing structures to withstand seismic loads. The seismic design philosophy, which tries to balance a structure's strength, ductility, and energy dissipation capability, includes the response reduction factor as a key component. Bangladesh National Building Code BNBC (2020) incorporates response reduction factors. This code defined guidelines and standards for building design in Bangladesh's earthquake-prone areas. The seismic hazard level, occupancy type, and relevance of the structure are just a few examples of the aspects that the response reduction factor takes into account while making sure that structures are designed to satisfy specified performance requirements. By taking into account the nonlinear behaviour of materials and elements, nonlinear static analysis, sometimes called pushover analysis, is necessary for structural engineering to provide a thorough knowledge of a structure's behaviour under lateral loads, notably during seismic occurrences. The seismic response reduction factor (R-factor), which pushover analysis uses to account for the expected inelastic behaviour of structures during an earthquake, is an important parameter. It displays a structure's capacity for energy dissipation and ductile deformation. To get the target displacement profile for the pushover analysis, the response reduction factor is usually applied to the elastic response spectrum.

Based on the selected reduction factor, the structure's response is altered as the analysis moves through increasing lateral force levels. This enables engineers to model the structure's anticipated nonlinear behaviour under seismic loads. R-factor is a useful tool for seismic design and evaluation since it creates a more accurate picture of the structure's performance, particularly in the inelastic range. An unduly conservative number of seismic response reduction factor may incur additional costs, while an extremely low value of it may jeopardize the structural integrity. The primary objective of this study is to use nonlinear pushover analysis to examine how the seismic response reduction factor affects the ductility and strength of RC frames designed and detailed as per BNBC (2020) that are subjected to seismic forces. This study focuses on assessing the R values for moment-resisting RC frames and the variation of over-strength and ductility factors in RC buildings with different seismic zones and the number of stories.

2. METHODOLOGY

A commercially available software ETABS 17.0.1 was used to model RC buildings. Then the models were analysed and designed as per BNBC 2020 code. Pushover analysis was carried out for that model with the help of ETABS.

2.1 Pushover Analysis

The capacity curve must be developed in order to perform the nonlinear pushover analysis. The building's nonlinear analysis yielded the capacity curve. A capacity curve for the building is created during the incremental nonlinear static analysis procedure. This capacity curve is just a plot of the building's total lateral seismic demand, represented by the letter "V," at different loading increments versus the building's lateral deflection at the roof level under that applied lateral force. This capacity curve would be a straight line with a slope equal to the global stiffness of the building if the building had infinite linear capacity.

The capacity curve, which shows the progressive degradation in structural stiffness that happens as the building is subjected to increased lateral displacement, yielding, and damage, usually consists of a series of straight-line segments with decreasing slopes because real buildings do not have infinite linear capacities. "d" denotes the secant or "effective" stiffness of the building when pushed laterally to that displacement. It is calculated as the slope of a straight line drawn from the plot's origin to a point on the curve at any lateral displacement (ATC-40,1996). Figure 1 displays a typical capacity curve for a fictitious building.

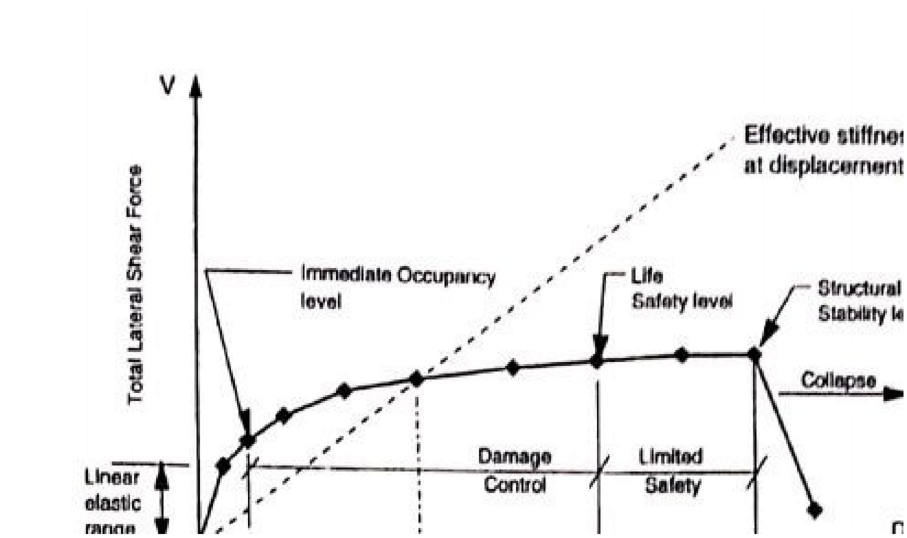

Figure 1: Normalized Capacity Curve (Taken from ATC-40, 1996)

In Fig. 1, Important events that have occurred in the building's lateral response history are represented by the discrete points denoted by the symbol "*". As illustrated in figure 1, there are three performance levels: Immediate Occupancy (IO), Life Safety (LS), and Structural Stability (SS).

2.2 Nonlinear Hinge Properties

The study deals with assigning auto hinges to RC frame elements. M3 hinges are applied on both sides of beam elements and P-M2-M3 hinges are applied on both sides of column elements as per ASCE 41-17, a standard developed by the American Society of Civil Engineers (ASCE) guideline. In ETABS, hinges are used to simulate the nonlinear behaviour of structural elements at their connections or critical sections, such as beams, columns, and walls.

2.3 Response Reduction Factor R

The original pushover curve must be transformed into a bilinear curve in order to compute over-strength and ductility factor. The product of these two factors is called response modification factor. The pushover curves are bi-linearized using a four-parameter power model. Richard and Abbot (1975) first proposed this model to simulate the elastic-plastic stress-strain relationship.

As a bilinear curve, the pushover curve was idealized so that, up to the final displacement, the areas under both curves were roughly equal (Newmark, Hall, 1982). The bilinear approximation of an idealized pushover curve is depicted in Figure 2, where V_u represents the maximum base shear capacity, V_d the design base shear, Δ_u the ultimate displacement, Δ_y the yield displacement, V_y the corresponding yield base shear from the bilinear approximation. (Sanches, Tao, Fathieh, Mercan,2021).

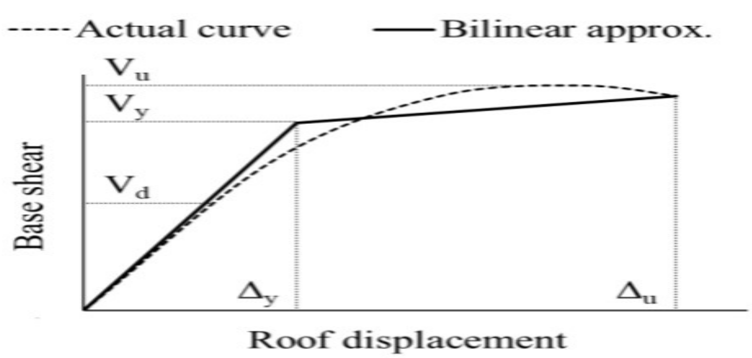

Figure 2: Pushover curve in general and bilinear approximation (Sanches, Tao, Fathieh, Mercan ,2021)

The building does not respond entirely elastically in the event of a strong earthquake, even though lateral forces are applied in accordance with the building's elastic vibration modes. In such cases, there is usually a significant amount of nonlinear deformation experienced by the building. As a result, it is evident that structures are capable of withstanding earthquakes stronger than the ones for which they were designed. The building acquired reserve strength and ductility, which is the reason behind this. Usually, the product of the over-strength factor and ductility yields the response modification factor. The over-strength factor and the ductility factor are estimated using the pushover curve that results from the pushover analyses. By multiplying these values, the response modification factor R is found.

The SMRF building's R factor is eight, as stated by BNBC 2020. However, a variety of structural properties influence the response reduction factor (R), which is represented by equation (1). These properties include ductility, over-strength, damping, redundancy.

$$R = R_s \times R_\mu \times R_R \times R_\zeta \qquad (1)$$

Where, R_s is the over-strength factor, R_μ is the ductility factor, R_ζ is the damping factor, and R_R is the redundancy factor.

2.3.1 Bilinear Idealization of Pushover Curve

First, a point needs to be assumed as the yield point (V_y, Δ_y) based on FEMA-356 (2000). Two parts make up a bi-linear curve: the elastic and post-elastic portions which are depicted in figure 3. The elastic portion will then be obtained by joining the origin to the presumptive yield point. However, the point where the real curve and elastic line intersect must have $0.6V_y$, where V_y is shear at the yield point shown in figure 3. It is a prerequisite for changing the assumed yield point. The next step involves determining the corresponding base shear V_u and the maximum displacement Δ_u. Since the maximum load has been reached, it has been nearly constant in the last iterations. Every subsequent iteration sees an increase in the corresponding displacement. In order to ensure that the failure occurs at the beginning of the plastic area and that it reaches a sufficient level of safety, it is advised to select a step with a relatively low displacement change. The yield point is linked to this maximum displacement point. The real curve and the area under the bi-linear curve need to roughly match. If not, the assumed yield point needs to be modified in order to obtain it.

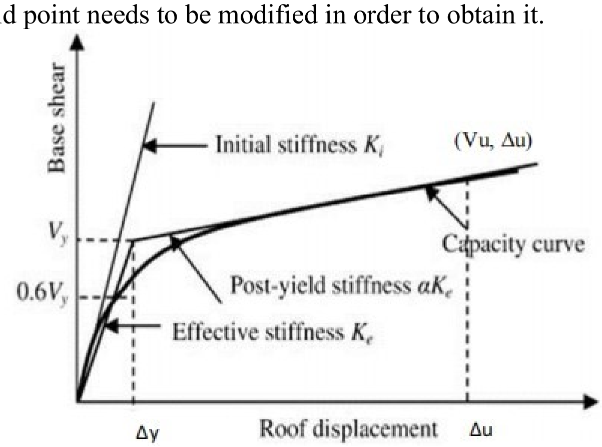

Figure 3: Bi-linear idealization of pushover curve (FEMA-356,2000)

2.3.2 Over-strength Factor

In seismic design, the over-strength factor is a parameter that takes into account the probability that a structure will withstand an earthquake and still hold up without collapsing. To improve the dependability and safety of structures, it is included in seismic design codes and guidelines. Greater ductility and energy dissipation are essential for reducing seismic damage, and a higher over-strength factor suggests these qualities. From figure 2, the structural system's intrinsic over-strength is represented by R_s, which is computed by dividing the design shear (V_d) by the maximum/ultimate base shear (V_u) expressed by equation (2). (ASCE 7-16, 2017; Charney F.A., Bertero V.V.,1982)

$R_s = \frac{V_u}{V_d}$ (2)

In this case, the maximum base shear is denoted by V_u and the design shear by V_d.

In accordance with BNBC 2020, the seismic design base shear force in a specific direction shall be calculated using equation (3).

$V = S_a \times W$ (3)

where S_a denotes the coefficient of lateral seismic force. According to BNBC 2020, equation (4) expresses the design spectral acceleration corresponding to building time period T. W, the sum of the dead load and 25% of the live load, represents the building's overall seismic weight.

$S_a = \frac{2}{3} \times \frac{ZIC_s}{R}$ (4)

Z represents the seismic zone coefficient, I stands for the structure importance factor, and Cs is the normalized acceleration response spectrum. The response spectrum is dependent on the soil type and building time period T.

2.3.3 Ductility Factor

The ability of a structure to tolerate inelastic deformations without experiencing a significant loss of strength is measured by a metric called the ductility factor in seismic design. Since it reduces the chance of abrupt failure, it is a desirable feature in seismic design and enables a structure to absorb and dissipate energy during an earthquake. The Rμ factor takes into account the effects of ductility. Over the last thirty years, a lot of work has been done to calculate the ductility factor based on SDOF systems that are exposed to various types of ground motions. Several notable and often cited works are among them, including those by Newmark and Hall (1982), Riddell and Newmark (1979), Vidic et al. (1992), Krawinkler and Nassar (1992), and Peter Fajfar (2021). The R–μ–T relationships created by Peter Fajfar (2021) are used in this study. According to Fajfar (2021), equations (5), (6), and (7) can be used to express the ductility factor.

$R_\mu = (\mu - 1)\frac{T}{T_c} + 1 \quad \text{for } T < T_c$ (5)

$R_\mu = \mu \quad \text{for } T \geq T_c$ (6)

$\mu = \frac{D_u}{D_y}$ (7)

Where, R_μ denotes ductility factor, T is the time period of structure, T_c is the upper limit of the period of the constant spectral acceleration branch, μ is the ductility deformation, D_y is the displacement at yield base shear, D_u is the displacement at ultimate base shear.

Equations (5) and (6) predict a linear relationship between the R_μ factor and the period T in the short time period, as shown in Figure 4, and take into account the equal displacement rule in the medium and long time period. (Adopted from Fajfar, 2021)

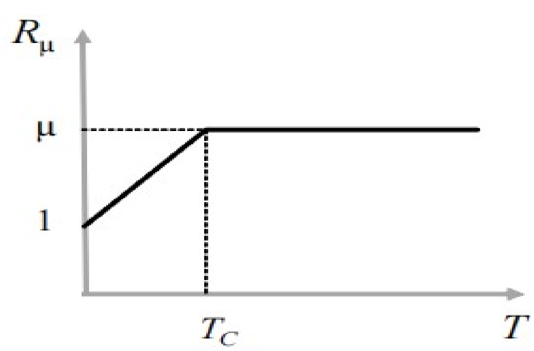

Figure 4: Normalized spectrum for R_μ factor (Adopted from Fajfar P. ,2021)

One benefit of equations (5) and (6) is the taking into account the frequency of the vibration of ground (calculated by Tc), which is typically affected by the nature of the earthquake and the soil conditions. The majority of the ductility factor formulas proposed by others lack this feature (Fajfar, 2021).

The redundancy factor, R_R, is taken into account to be 1.0 as per the American code ASCE 7-05 (2005). R_ζ is the damping factor that is most significant when connecting damping devices in the structure; otherwise, the factor should be taken as 1.0 (1999). As a result, depending on each of these characteristics, the response reduction factor value changes.

3. ILLUSTRATIONS

3.1 Description of The Frame Structure

The buildings identified in this section as G+3, G+6, G+9, and G+12 are thought to be situated in seismic zones II, III, and IV. The ETABS v.17 software is used for the pushover analysis and modelling of the structural systems. The building falls under the "Residential" operational category. This is a three-by-three bay structure with X- and Y-directional spans of 7.62 and 6.10 meters, respectively. Three meters is the standard story height. In seismic zone III, the G+6 building's layout plan and typical elevation are depicted in Figure 5. Every story level has a fixed floor diaphragm. The reinforcement ratio of the columns is within 1% to 4%. Every support is regarded as permanent support. It is assumed that slabs don't carry any moment. The structure is regarded as a special moment-resisting frame. The compressive strength of concrete and the yield strength of steel are considered of 27.6 MPa and 414 MPa, respectively. The value of R=8 has been considered for SMRF building as per BNBC 2020. The live load, floor finish load and partition wall load are taken to be 2,1.20 and 2.87 kN/m^2. The wind load is disregarded in favour of analysing the building's or structure's structural performance under lateral earthquake load alone. The earthquake load is calculated as per BNBC 2020. The following factors and coefficients are used for seismic zone III-

Response modification factor, R= 8 (for SMRF structure)
Zone factor, Z = 0.28
Importance factor, I= 1.00
Occupancy category = II
Soil type = SC
Factor for soil, S = 1.15
Table 1 lists the dimensions of the columns and beams.

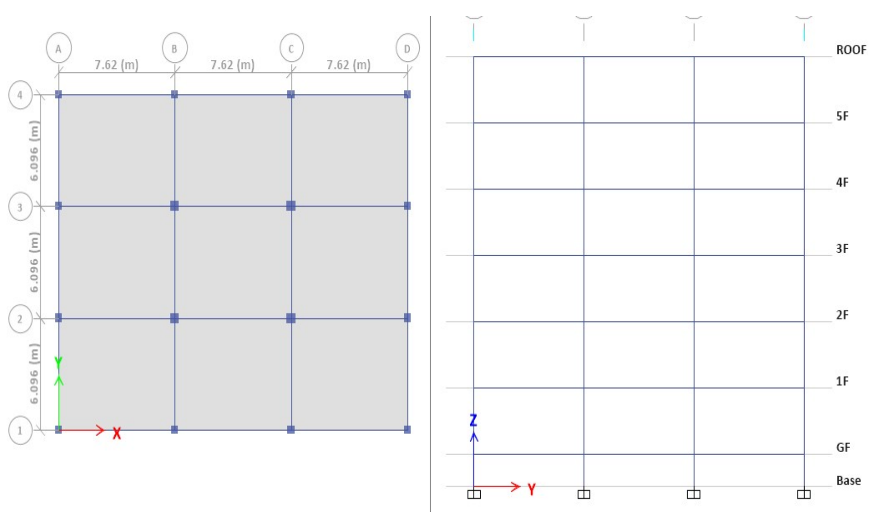

Figure 5: Layout plan and elevation of RC structure

Table 1: The dimensions of RC sections

Frame	Members	Dimensions mm	Frame	Members	Dimensions mm
G+3	**Beam**	300×400	G+9	**Beam**	300×500
	Column			**Column**	
	Internal	475×475		Internal	675×675
	External	380×380		External	550×550
G+6	**Beam**	300×450	G+12	**Beam**	350×550
	Column			**Column**	
	Internal	550×550		Internal	775×775
	External	450×450		External	650×650

Pushover analysis is used in this study's execution to obtain the structure's nonlinear response. The overstrength and ductile capacity of the structures are estimated using pushover analysis. Since the NSPA is used to evaluate the R values, it is imperative that all members take part in the analysis for effective outcomes. The idea of auto hinges will emerge because non-linear behaviour is crucial. Auto moment M3 hinges are considered and auto axial P-M2-M3 hinges are considered at each end of beams and columns as per ASCE 41-17 (2017) for the building.

3.2 Evaluation of Response Reduction Factor R

The structural elements are examined for their hinges after the pushover study is finished and the nonlinear pushover curve is then derived. Figure 6 displays the push over curves for four structures in zone IV (SMRF).

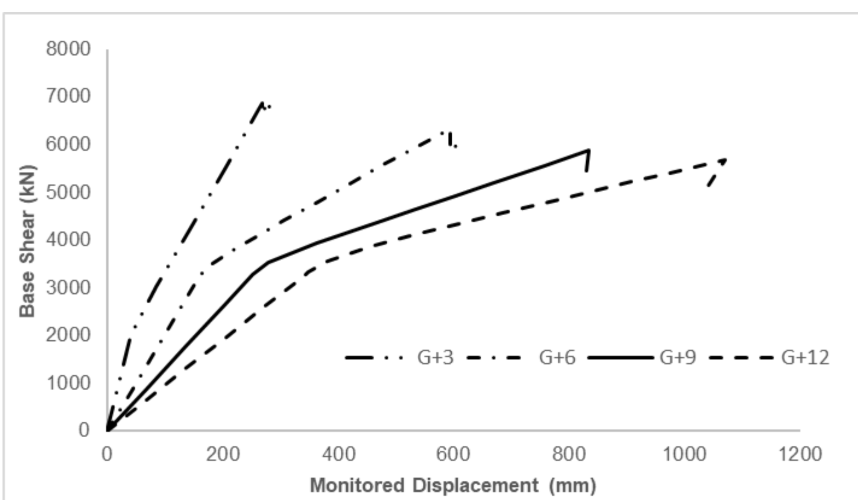

Figure 6: Pushover curves of SMRF buildings at seismic zone IV

In order to get values for ultimate shear (V_u), yield displacement (D_y), yield shear (V_y) and ultimate displacement (D_u), this curve must be bi-linearized. The pushover curve is bi-linearized based on FEMA-356 (2000) and is used to evaluate ductility factor and over-strength factor. The product of these two factors is called response modification factor R. The bi-linearized pushover curve of G+6 building in seismic zone IV is depicted in figure 7.

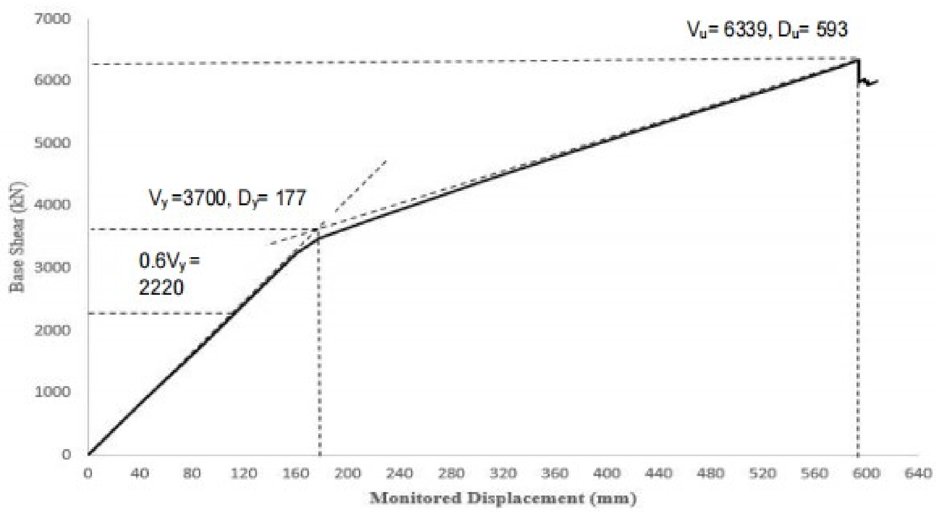

Figure 7: Bi-linearized pushover curve of G+6 building in seismic zone IV

First, a point is assumed as the yield point (V_y=3700 kN, D_y= 177 mm). The elastic portion is obtained by joining the origin to the presumptive yield point. However, the point where the real curve and elastic line intersect must have $0.6V_y$ and from figure 7, the elastic line intersects real curve at $0.6V_y$ = 2220 kN. From figure 7, the maximum base shear is V_u = 6339 kN and maximum displacement is D_u = 593 mm. The yield point is connected to this maximum displacement point. The real curve area and the area under the bi-linear curve approximately matches.

Height of Building (m): 18 meters

As per BNBC 2020,

Site class= SC

Factor for soil, S= 1.15

T_B=0.20, T_C=0.60, T_D=2

Building time period $T= C_t \times (h_n)^m$

$= 0.0466 \times (18)^{0.9} = 0.628$

So, $T_C \leq T \leq T_D$

Viscous damping ratio, ζ= 5%

$\eta=\sqrt{\frac{10}{5+\zeta}}=\sqrt{\frac{10}{5+5}}=1$

Response reduction factor, R: 8 (For SMRF Structure)

Importance factor: 1.0

Seismic zone: 4; Z= 0.36

Normalized acceleration response spectrum, $C_s= 2.5\times S\times \eta \times\frac{T_C}{T}$ for $T_C \leq T \leq T_D$

$= 2.5\times1.15\times1\times\frac{0.6}{0.628} = 2.74$

Using equation (4), Design spectral acceleration $Sa= \frac{2}{3}\times\frac{ZIC_s}{R} = \frac{2}{3}\times\frac{0.36\times1\times2.74}{8} = 0.08$

From ETABS data, seismic weight W = Dead Load+ Partition wall load+ Floor finish load+ 0.25Live load

= 18830+8407+3503+(0.25×5605)

= 32141 kN

Using equation (3), design base shear, $V_d = Sa \times W = 0.08 \times 32141$ =2571 kN

Using equation (2), over-strength factor, $R_s = \frac{Vu}{Vd} = \frac{6339}{2571} = 2.46$
Since, T=0.628 > T_c = 0.6 and using equation (6) and (7),
Ductility factor, $R\mu = \frac{Du}{Dy} = \frac{593}{177} = 3.35$
Response reduction factor, $R = R_s \times R\mu = 2.46 \times 3.35 = 8.24$

After doing the calculations for the buildings, table 2 represents the values of R_S, Rμ and R.

Table 2: Estimated values of over-strength factor, ductility factor and R factor

Zone	**Story**	**Time Period**	**R_s**	**Rμ**	**R**
II	G+3	0.337	6.48	1.28	8.29
	G+6	0.628	3.34	2.53	8.45
	G+9	0.905	3.01	2.89	8.7
	G+12	1.172	2.83	3.15	8.91
III	G+3	0.337	5	1.64	8.2
	G+6	0.628	2.63	3.16	8.32
	G+9	0.905	2.47	3.51	8.66
	G+12	1.172	2.27	3.91	8.87
IV	G+3	0.337	4.36	1.87	8.15
	G+6	0.628	2.46	3.35	8.24
	G+9	0.905	2.24	3.84	8.6
	G+12	1.172	2.1	4.8	8.82

4. RESULTS AND DISCUSSIONS

The pushover analysis was performed and many factors were taken into account for the study in order to effectively comprehend the basics of response modification factor. The elements taken into account are the zone factor and the time period to draw further conclusions.

4.1 Over-strength Factor

4.1.1 Effect of The Time-period

Time period of building increases with the increase of number of stories. In that case, in figure 8.a, as the time period or height of the building increased, the over-strength factors decreased in each zone. While increasing the number of stories, the design base shear increases and the ultimate base shear decreases. As a result, it gradually decreases the over-strength factor. Higher values of over-strength factors are seen in special moment-resistant frames of buildings analysed for a smaller seismic zone. All three seismic zones represented the similar effects of heights on the over-strength factor.

4.1.2 Effect of Zone Factor

The structure's response varies for different seismic zone. It is observed in figure 8.b that the zone II has higher over-strength factor. With the increase of seismic zone, the over-strength factor reduces and it reflects that the zone factor has an effect on over-strength factor which is inversely proportional. All four (G+3, G+6, G+9 and G+12) buildings represented the similar effects of seismic zones on the over-strength factor.

4.2 Ductility Factor

4.2.1 Effect of The Time Period

For the short time period buildings in seismic zones III and IV, as shown in figure 8.c, the ductility factor exhibits minimal variation. However, as the natural time period of the structures lengthens, as shown in figure 8.c, the building's ductility increases. The building frames which are modelled and analyzed for low seismic zones give lower ductility than those analyzed for higher seismic zones.

4.2.2 Effect of Zone Factor

The variation of the ductility factor for seismic zones is studied in figure 8.d. It is noted that the ductility increased as the zone factor increased for all four types of buildings. In figure 8.d, there is little variation of ductility factor in lower seismic zones for G+6, G+9 and G+12 buildings. But with the increase of seismic zones, the ductility gradually increased.

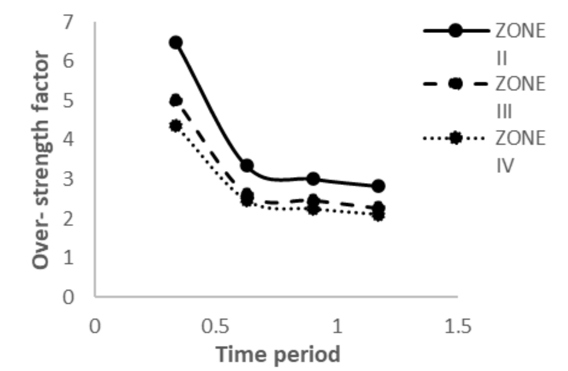

Figure 8.a: Effect of time period on over-strength factor

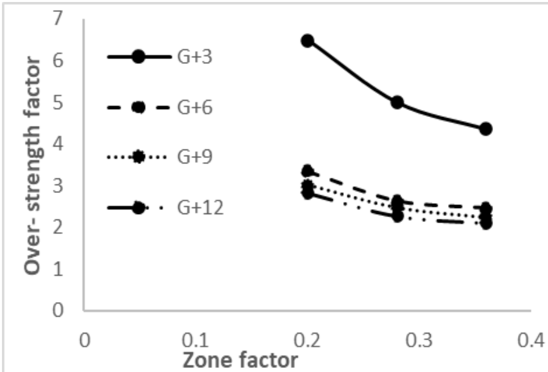

Figure 8.b: Effect of zone factor on over-strength factor

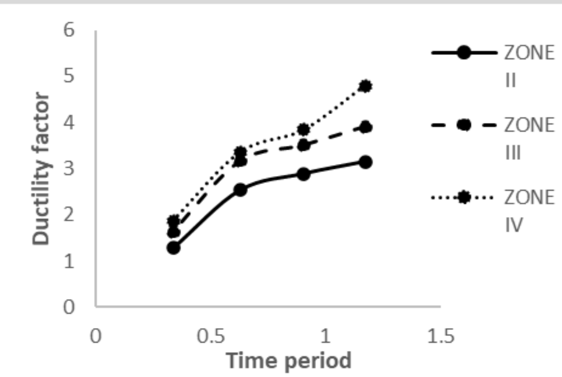

Figure 8.c: Effect of time period on ductility factor...

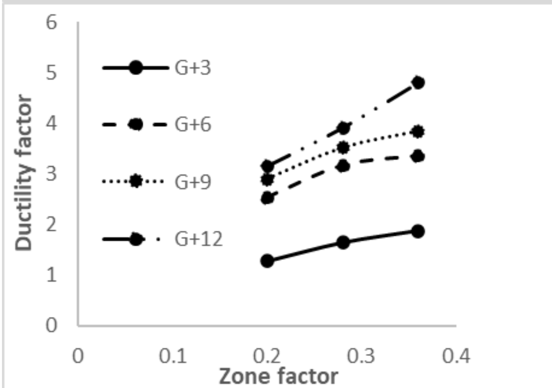

Figure 8.d: Effect of zone factor on ductility factor...

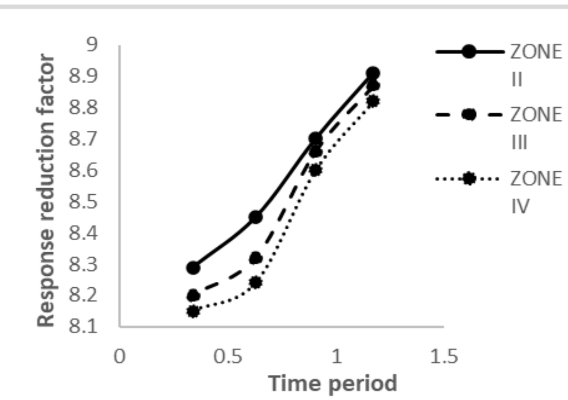

Figure 8.e: Effect of time period on response reduction factor...

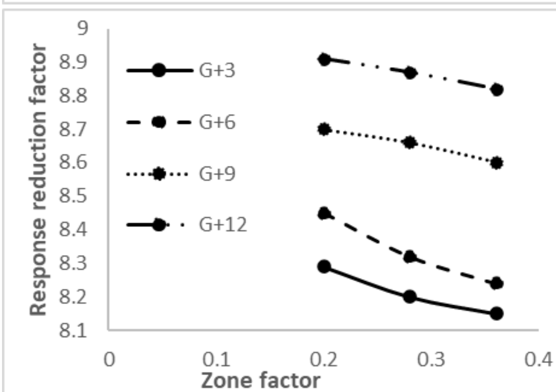

Figure 8.f: Effect of zone factor on response reduction factor

4.3 Response Reduction Factor R

4.3.1 Effect of The Time Period

Seismic response modification factors seem to differ with time period of the structure. BNBC (2020) specified value of response modification factor R for SMRF building is 8. The values of R for the buildings with different time periods varied within the range of 8.1 to 8.9 in figure 8.e. The values of R are 1.25% to 11.25% higher than the code specified value. These buildings can be considered conservative as their response reduction factor (R) is higher than the value required by BNBC 2020, indicating that it was built with more seismic resistance than the code specifies.

4.3.2 Effect of Zone Factor

Seismic zones affect the overall performance of the structure. The response reduction factor's value falls as the seismic zone factor rises. But all values are in range of 8.1 to 8.9 in figure 8.f. which are higher than BNBC (2020) specified value of 8. So, these values of R are 1.25% to 11.25% higher than the code specified value. Overall, these buildings can be considered conservative with having more margin of safety.

5. CONCLUSIONS

The following results are obtained for G+3, G+6, G+9 and G+12 SMRF buildings which are located in seismic zone II, III and IV. These buildings have been analysed by nonlinear static pushover analysis for assessment of ductility factors, over-strength factors and response modification factors R.

- Based on the design and ultimate base shear, the over-strength factor changes according to seismic zones and the natural time of the building frames.
- The over-strength factor for SMRF frames also lowers as the structure's time period lengthens. It indicates that low rise SMRF buildings have higher over-strength values than the taller SMRF buildings.
- The buildings that are examined for lower seismic hazard zones offer significantly higher over-strength. It also shows that higher over-strength helps to minimize structural damage of buildings in lower seismic zones. On the other hand, the over-strength value decreases with increase of seismic zone factor values.
- The ductility of the building enhanced with the buildings' natural time period. Taller buildings with greater ductility are better able to absorb and dissipate seismic energy, which lessens the overall effect of seismic forces on the structure. It is also noted that the ductility increased as the zone factor increased for all four types of buildings.
- The time period increased when the number of stories increased from 3 to 12 and the R factor increased slightly. As increase of number of stories give different values of the time period and R factor, it indicates that R factor does not have a constant value. There is a relationship between R factor and time period of the building.
- The seismic zones affect the overall performance of the structure. As the seismic zone increases, the overall seismic response modification factor, which is dependent on ductility and over-strength components, drops. A decline in the response reduction factor in higher seismic zones suggests that the structure's capacity to release energy through inelastic deformations has lowered, which has resulted in a decline in the inelastic performance of the structure.
- All values of R are higher than BNBC (2020) specified value for SMRF building of 8. So, these buildings can be considered more conservative than required by the code. It also indicates that the structure will provide better performance in terms of seismic resistance than the requirements set by the code.

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