AN EXPERIMENTAL STUDY ON THE PARTICLE SIZE GRADING, STRENGTH, AND DEFORMATION CHARACTERISTICS OF RAILWAY BALLASTS

Md Hamidur Rahman[*1] **, Abu Yousuf Mohammad Shamim**[2]**, and Kimitoshi Hayano**[3]

[*1] *Additional Chief Engineer(Bridge),East, Bangladesh Railway, Bangladesh,* hrahman_2000@yahoo.com
[2]*Additinal Chief Engineer(Track and Works), PBRLP, Bangladesh Railway,* aymsamim@gmail.com
[3]*Faculty of Urban Innovation, Yokohama National University, Yokohama, Japan,*
hayano-kimitoshi-hg@ynu.ac.jp

***Corresponding Author**

ABSTRACT

Railway ballast is uniformly graded coarse aggregate placed between and immediately underneath the sleepers to provide drainage and structural support for the load coming from the moving trains. Ballast is one of the major sources of construction and maintenance costs for railway tracks. Ballast particle grading critically influences the mechanical behaviour of ballast. In this research, a comparative study has been made to assess the grading performance stipulated in the Bangladesh Railway Ballast Specification (BRS). For this study, two gradings (BR-1 & BR-2) are selected from the boundary of BRS, one grading is selected from Japanese Standard (JP-1), and another grading is selected from the American Railway Engineering and Maintenance-of-Way, AREMA-24 (AREMA-24(1)). The experimental study was carried out on 1/5th scaled clean and dry ballast. A series of both monotonic and cyclic triaxial tests were conducted. The selected sample's physical properties and the triaxial test results are analyzed rigorously. The study found that the uniformity coefficient has a major impact on increasing the strength and durability of ballast and reducing strain and ballast breakage. It is also found that ballast breakage increases with increasing the particles' median (D_{50}) size. It is observed that stress history significantly impacts reducing strain in ballast and its impact increases with the ratio between the previous stress history and currently applied stress. Moreover, the study reveals that there is no definite relationship between the coefficient of uniformity and stiffness. Finally, after comparison, it is found that as per the test condition of the fresh ballast, the performance of BR-2 is better than other gradings and is followed by BR-1, AREMA-24(1), and JP-1 respectively.

Keywords: *Ballast Grading, Monotonic and Cyclic triaxial tests, Coefficient of Uniformity, Ballast Breakage Index, Permanent Strain, and Deviator Stress.*

1. INTRODUCTION

Ballast is the crushed granular material which is placed in the top layer of the substructure. These coarse-grained materials are used to support and confine the sleepers, and to minimize any vertical and lateral movement transferred to the sleepers, and hence retain track position. The ballast material also reduces sleeper pressures and distributes it to the underlying materials, e.g. sub-ballast and subgrade. The ballast also provides a certain amount of resilience as well as energy absorption for the rail track. The provision of large voids in the ballast layer facilitates the storage of fouling materials as well as the immediate draining of water from the track. Ballast is also important for providing the fastest and most economical method of restoring track geometry, especially in a subgrade failure situation. However, ballast is also one of the main sources of track geometry deterioration. The ballast layer accounts for up to 65% of the overall track stability on the condition that the ballast layer is stabilized and compacted as reported by Zakeri, Esmaeili, Kasraei and Bakhtiary (2014). On the other hand, it is the major cause of track settlement compared to sub-ballast and subgrade(Selig & Waters, 1994). So to reduce the vertical track settlement and ensure intended track stability, emphasis must be placed on the ballast. Maintenance work to re-level the railway track, namely tamping, is often costly, destructive and disruptive. Lower train speed is expected after tamping (to allow the track to settle) and this often disrupts the train schedules. It should be mentioned that temping of ballast is also one of the major sources of ballast degradation. So, frequent temping spurs the need for early recoupment of railway ballast.

Characteristics of railway ballast define its effectiveness in service and railway ballast specification determines the characteristics of railway ballast. A good railway ballast specification ensures stability and drainage to the railway track. It increases the interval between the requirement of temping and recoupment of ballast in the railway track. Which consequently reduces the maintenance cost of the railway track i.e. life cycle cost of the railway track. Railway ballast grading is one of the most important elements of ballast specification.

The distribution of particle sizes has an obvious and significant influence on the deformation behaviour of the railway track (Jeffs and Tew, 1991). Several researchers have studied the effects of particle gradation on the strength and deformation of the aggregates. Thom and Brown (1988), conducted a series of repeated load triaxial tests on crushed dolomite with similar maximum particle sizes, but varying the gradation from well-graded to uniform. According to their results elastic shear stiffness and permeability increase as the uniformity increases. On the other hand, the density and friction angle decreases with uniformity. Raymond(1985) states that single-sized ballasts have larger void volumes than broadly graded ballasts. Thus, where the production of fines from aggregate breakdown under train loads is a major source of contamination, single-sized ballasts are generally preferred. However, well-graded or broadly graded ballast is stronger due to its void ratio being smaller than uniform ballast (Jeffs and Tew, 1991; Raymond,1985; Marshal, 1967).On the other hand, Raymond and Diyaljee (1979) stated that a more well-graded ballast has proved to perform, minimizing permanent deformations, as well as or better than uniformly sized ballast within the same size range. Roenfeldt (1980) [10] conducted repeated load triaxial tests on limestone ballast. One set of specimens had a narrow range of particle size ($C_u = 1.14$); the other set had a broader grading ($C_u = 4.1$) but a slightly smaller mean size. The cumulative plastic strain for the uniform ballast was almost double than that of more broadly graded ballast. Furthermore, the particle degradation for the uniform ballast was four to five times greater than for the more broadly graded ballast. Klugar, 1978 [11] reported that replacing a small amount (<15%) of large particles in a ballast specimen with smaller particles increased the friction angle in the shear box test. In contrast, replacing a greater amount (>20%) caused a significant reduction in friction angle mobilized at a given displacement.To optimize the distribution of particles, Skoglund (2002) compared a railway ballast material (gabbro) with a typical grading (following the Norwegian requirements) with another specimen made with a slightly more dense gradation, where the amount of smaller grains exceeded the current specification. Skoglund found that the more-dense grading did not significantly change the permanent deformation.
Triaxial tests by Marachi, Chan and Seed(1972) showed that strength increases as particle size decreases. However, Dunn and Bora(1972) in triaxial tests previously conducted found that shear

strength increased with increasing particle size. Finally, triaxial test results and analysis by Vallerga, Seed, Monismith, and Cooper (1957), Holtz and Gibbs (1956) showed little effect of particle size on strength. Thus, the effect of particle size on strength is unclear.

Railway ballast specifications generally call for uniformly graded aggregates mainly to meet drainage requirements. Since ballast is expected to be a coarse, free-draining medium, the optimum gradation should ideally be between uniformly graded coarse aggregates that give almost instantaneous drainage and broadly graded aggregates that provide higher strength and less settlement at the expense of reduced drainage. According to Indraratna and Salim (2005), the optimum gradation should provide sufficient drainage capacity along with sufficient initial density, shear strength, resilient modulus and less settlement. There is, however, a need for more testing in this case to investigate the influence of grading over a broader span of grading variations.

In this research, a comparative study of the effect of ballast particle grading on the deformation characteristics will be made based on the laboratory test of railway ballast of BR Ballast Specification, Japanese Ballast Specification and AREMA-24 Ballast Specification. From this study effectiveness of BR Ballast Particle grading will be assessed against that of the Japanese and the AREMA-24. It will help determine the performance of existing railway ballast grading and choose new railway ballast grading.

2. PROPERTIES OF BALLAST PARTICLES FOR TESTING

In general, the behaviour of ballast is governed by characteristics of constituting particles, bulk properties of the granular assembly, loading characteristics and particle degradation, which is a combined effect of grain properties, aggregate characteristics and loading (Indraratna and Salim, 2005). Moreover, the quality of the ballast particle specified in each ballast specification is somewhat different. The ballast specification considered for this study is shown in Table 1.

Table 1: Recommended quality of ballast by different specifications

Characteristics test	BR Specification	JP Specification	AREMA Specification
Specific Gravity	2.6	Not Specified	2.6 min
LAA	35% max	27% max for shinkansen, 35% mas for other rail lines	35% max
Soundness Test	8% max	Not Specified	5% max
Absorption	1.5% max	3%	1% max

2.1 Ballast Grading for the Test

For the research purpose ballast grading of Bangladesh Railway Ballast Specification (BRS), Japanese Ballast Specification (JP) and American Railway Engineering and Maintenance of Way Association Ballast Specification (AREMA-24) have been used as shown in Table 2.

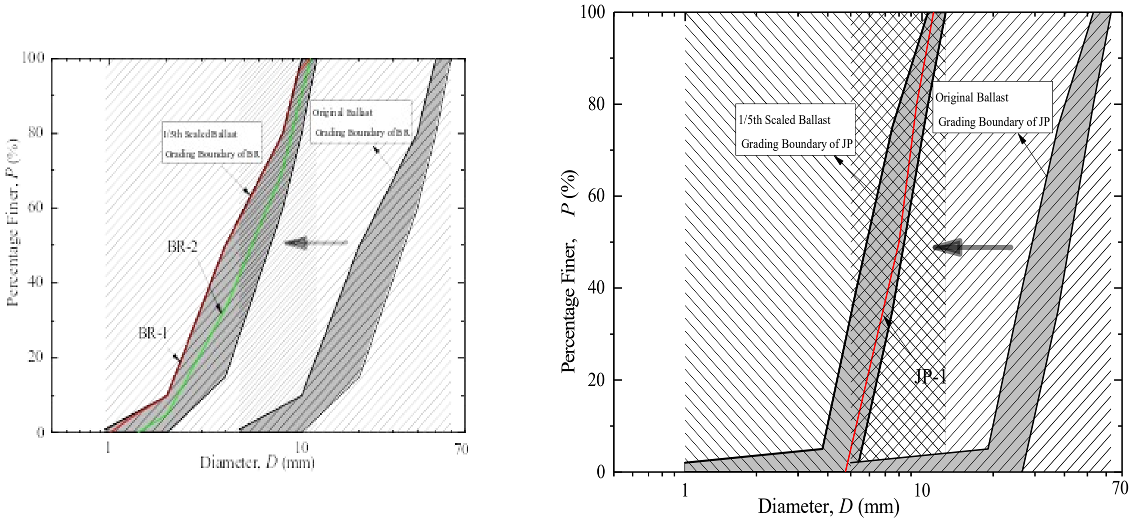
Table 2: Railway Ballast Grading of Different Specifications.

BR Specification		JP Specification		AREMA-24 Specification	
Sieve Size (mm)	% Finer by weight	Sieve Size (mm)	% Finer by weight	% Finer by weight	% Finer by weight
60	100	63	100	100	100
50	80-100	53	80-100	80-100	90-100
40	60-80	37.5	35-75	35-75	25-60
20	15-50	26.5	0-40	0-40	0-10
10	0-10	19	0-5	0-5	0-5
4.75	0-1	5	0-2	0-2	------

Due to the unavailability of large-scale triaxial machines, the test has been performed on the one-fifth (1/5th) scaled railway ballast. Roner (1985), Li, Hyslip, Sussmann, and Chrismer(2002) found that "parallel gradations" (i.e., which have a fixed gradation curve distribution shape but are scaled uniformly up or down in grain size) have essentially the same friction angle when compared at the same void ratio and also mentioned that the effect of particle size on the strength is unclear. Four different 1/5th scaled gradings (i.e. BR-1, BR-2, JP-1 & AREMA-24 (1)) from the above three Railway Ballast grading have been chosen for research purposes as shown in Table: 3 and Figure 1: 1/5th scaled BR-1,2, JP-1 and AREMA-24(1) ballast grading curve. The characteristics of scaled grading curves are shown in Table 4.

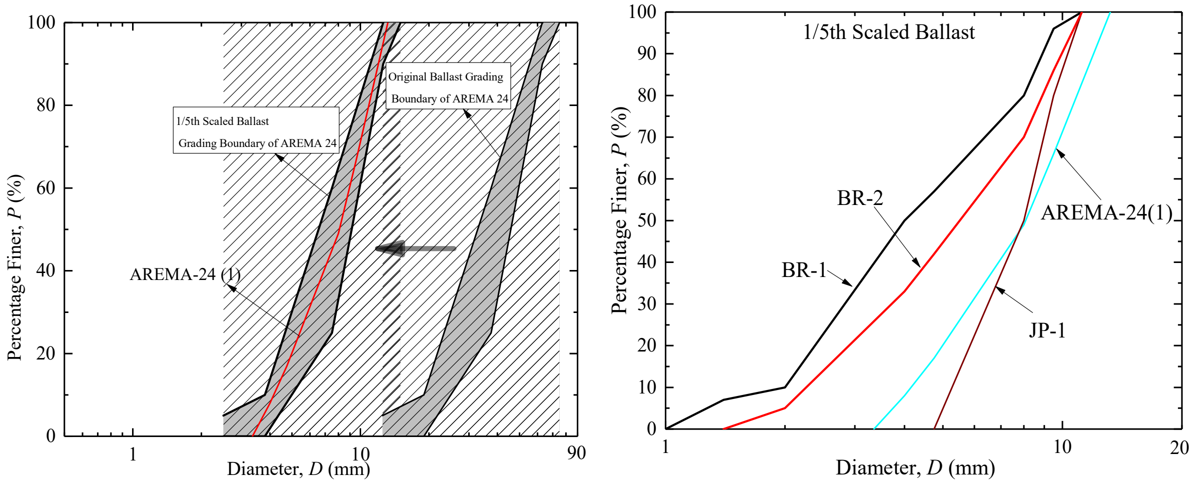
Table: 3 1/5th Scaled Ballast Grading.

Specification	BR-1	BR-2	JP-1	AREMA-24(1)
Size (mm)	% Finer	% Finer	% Finer	% Finer
13.2	-	-	-	100
11.2	100	100	100	83
9.5	96	86	80	66
8	80	70	50	49
4.75	57	42	0	17
4	50	33	-	8
3.35	40	25.85	-	0
2	10	5	-	-
1.4	5.23	0	-	-



a) BR-1,2 with BR grading boundary

b) JP-1 with Japan ballast grading boundary



a) AREMA-24(1) with grading boundary b) BR-1,2, JP-1 and AREMA-24(1)

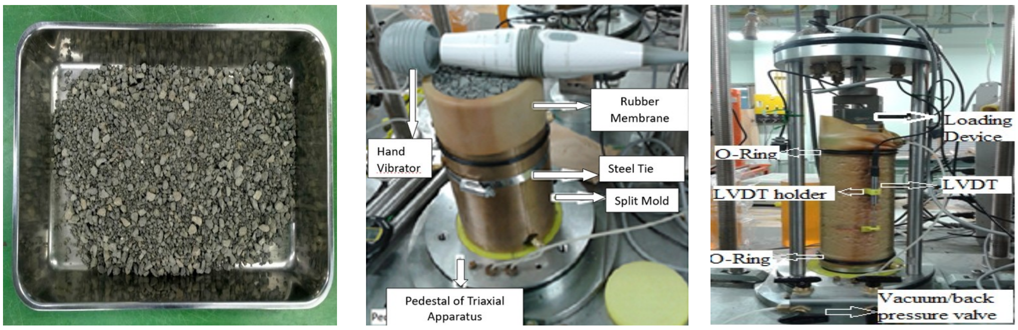
Figure 1: 1/5th scaled BR-1,2, JP-1 and AREMA-24(1) ballast grading curve with corresponding grading limits

Table 4: Characteristics of scaled grading curves

Ballast Specification	D_{max} (mm)	D_{min} (mm)	D_{60} (mm)	D_{50} (mm)	D_{30} (mm)	D_{10} (mm)	C_u	C_c
BR-1	11.2	1.00	5.08	4.00	2.83	2.00	2.54	0.79
BR-2	11.2	1.40	6.64	5.51	3.71	2.26	2.94	0.92
JP-1	11.2	4.75	8.47	8.00	6.49	5.27	1.61	0.94
AREMA- 24(1)	13.2	3.35	8.94	8.08	5.87	4.15	2.15	0.93

3. PREPARATION OF SPECIMEN FOR TEST AND TEST PROCESS

At first ballast particles have been washed and then dried for 48 hours. Afterwards, ballast particles are sieved and mixed to conform to the preselected BR-1, BR-2, JP-1 and AREMA-24(1) gradings (Figure 2(a)). Test specimens were prepared on the pedestal of the triaxial test apparatus. The diameter of the pedestal base is 100mm. To prepare the specimen for triaxial test a 1mm thick and 100mm diameter rubber membrane was placed around the pedestal. Silicon grease was applied between the base plate and the rubber membrane and two rubber O-rings were placed around the rubber membrane to make it watertight. Afterwards cylindrical split mould was placed to enclose the rubber membrane. The split mould was tightened by steel ribbon (tie) and the split mould was attached to an air vacuum machine, 60 kPa vacuum pressure was used to pull out the air between the split mould and the rubber membrane to stick the rubber membrane tightly with the split mould so that ballast could be poured into the membrane to make the specimen.



a) Mixed ballast b) Preparation of specimen c) Loaded Specimen

Figure 2: Mixed scaled ballast, preparation of specimen and loaded specimen

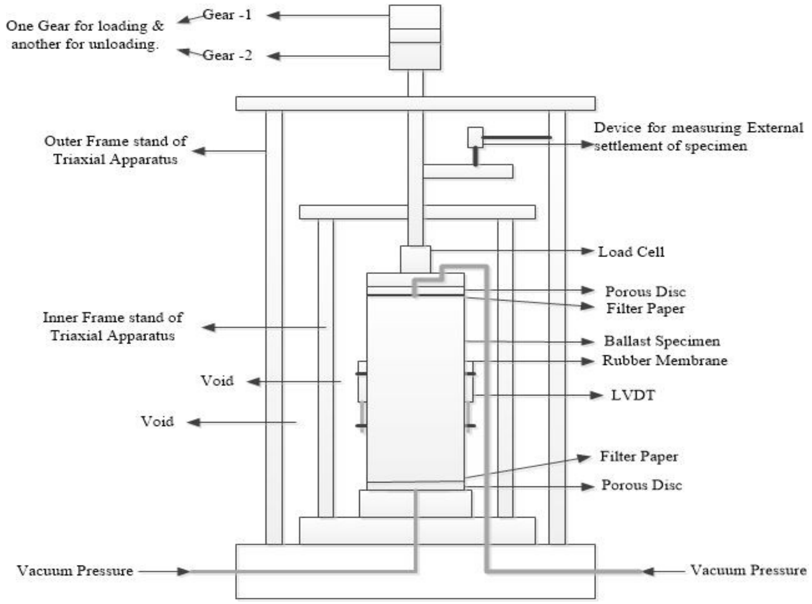


Figure 3: Schematic Diagram of the triaxial test

During the preparation of the specimen silicon gel were provided inside the rubber membrane at specific heights (4 points, 2 on each side) of the specimen to make the ballast stick together at that point. After preparation of the specimen supporting base of the LVDTs were attached at those points outside of the rubber membrane. This was done to avoid slippage between ballast and rubber membrane during the testing which might lead to erroneous LVDT readings (Figure 3).

3.1 Cyclic Triaxial Test Procedure

These tools had been calibrated beforehand and before conducting a new test each time the calibration parameters were checked. LabView software was used to control the test and to collect data automatically from the test. All the test was conducted by strain control procedure. The rate of loading for the triaxial test was 0.5%/min. Before applying the main load, the specimen was preloaded by 40 kPa cyclic deviator stress 300 times to consolidate. After the preloading, the specimen was repeatedly loaded in steps of 80 kPa for 1000 times. After the ending of each step, the 80 kPa load was increased until the failure of the specimen as shown in Table 5. After the test was completed the ballast of the specimen was collected and then sieve analysis was conducted to determine the change of gradation of specimen during the test.

Table 5: **Triaxial test stress and load cycles**

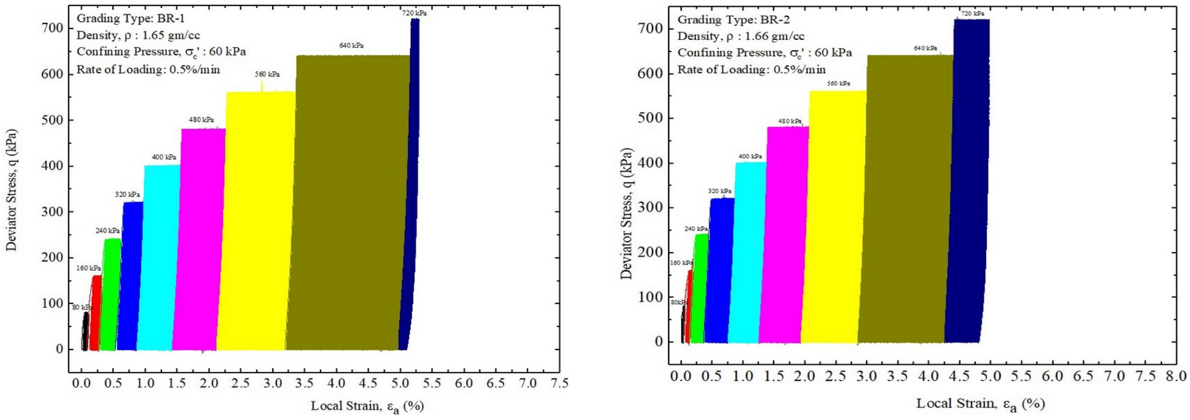
Serials	Stages	Deviator Stress(kPa)	Number of load cycles	Remarks
1	Stage-1	0-80	1000	At each step 80 kPa load is increased.
2	Stage-2	0-160	1000	
3	Stage-3	0-240	1000	
4	Stage-4	0-320	1000	
5	Stage-5	0-400	1000	
6	Stage-6	0-480	1000	
7	Stage-7	0-560	1000	
8	Stage-8	0-640	1000	
9	Stage-9	0-720	1000	

3.2 Monotonic Triaxial Test

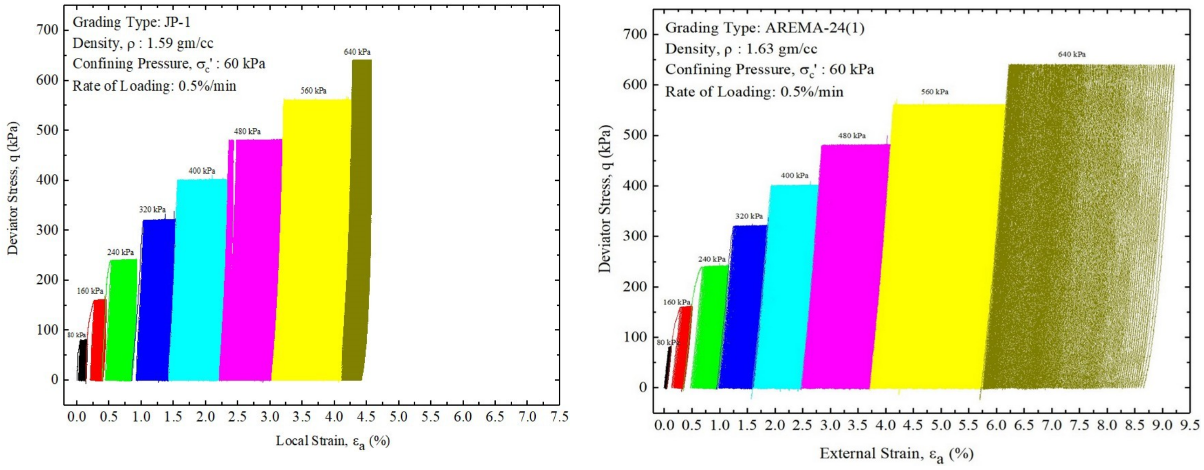
The same calibrated load cells, LVDTs were used for the monotonic triaxial test and calibration parameters were checked to every before conducting a test. The monotonic test was also conducted by strain control procedure and the loading rate was also same i.e. 0.5%/min. The test was continued till the failure of the specimen.

4. TEST RESULTS, ANALYSIS AND DISCUSSION

Test results of cyclic triaxial test for each specimen (i.e. BR-1, BR-2, JP-1 & AREMA 24 (1)) are shown graphically after processing shown in Figure 4 and Figure 5. Test results of the monotonic triaxial test for each specimen (i.e. BR-1, BR-2, JP-1 & AREMA 24 (1)) are graphically presented in Figure 6.

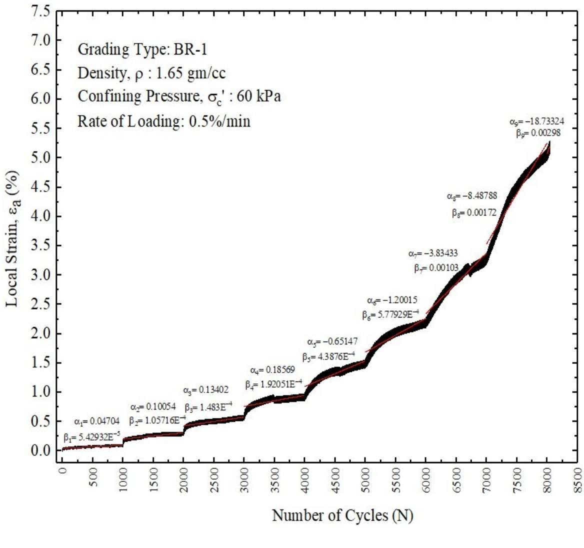


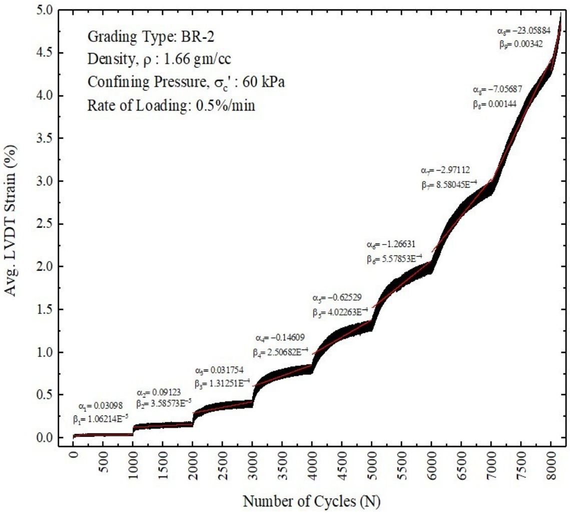
a) BR-1 b) BR-2



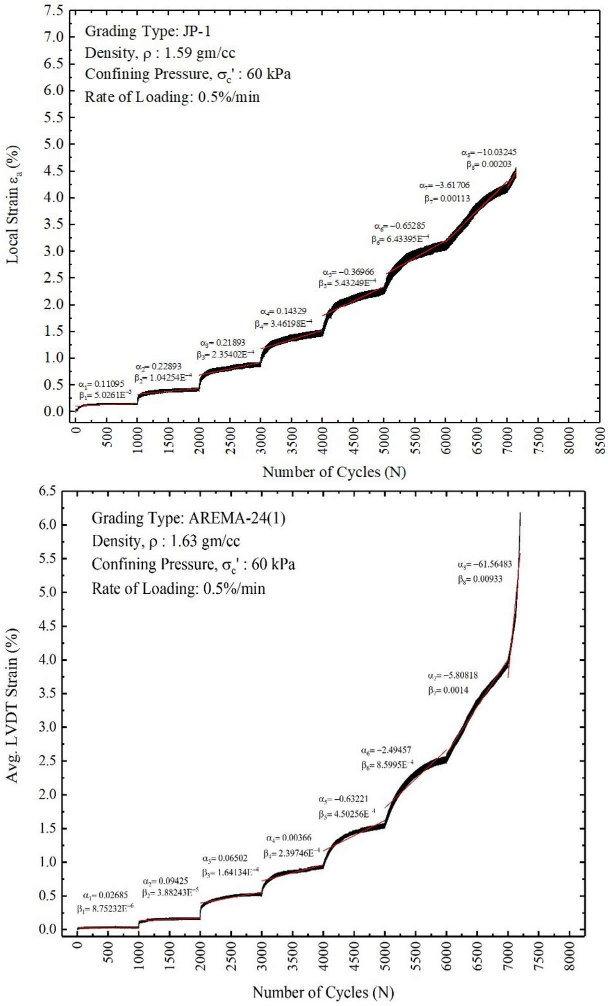
a) JP-1 b) AREMA-24(1)

Figure 4: Stress-strain relation of scaled samples BR-1,2, JP-1 and AREMA-24(1)





a) BR-1 b) BR-2



a) JP-1 b) AREMA-24(1)

Figure 5 Local Strain Vs Number of Load Cycles

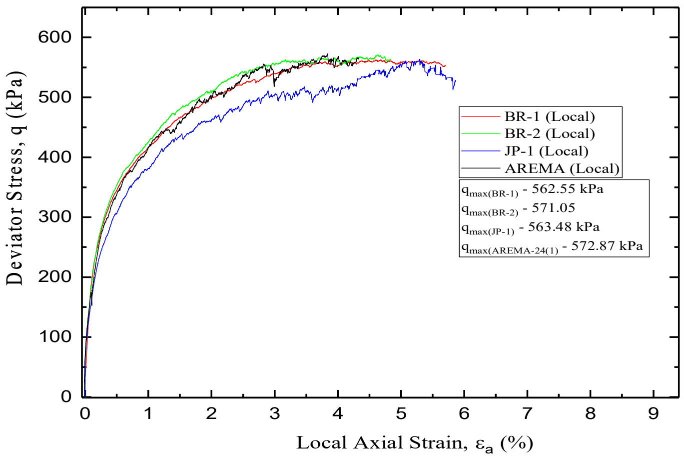


Figure 6 Deviator stress vs strain of monotonic test

4.1 Physical Properties

With increasing coefficient of uniformity (C_u) the range of sizes of particles increases that helps preparing denser specimen and consequently void ratio also decreases (Table 6)

Table 6 **Physical properties of the specimens.**

Specimen	Coefficient of Uniformity (C_u)	Dry density, (ρ) (g/cm^3)	Void Ratio (e)
BR-1	2.54	1.65	0.67
BR-2	2.94	1.66	0.66
JP-1	1.61	1.59	0.73
AREMA-24(1)	2.15	1.63	0.69

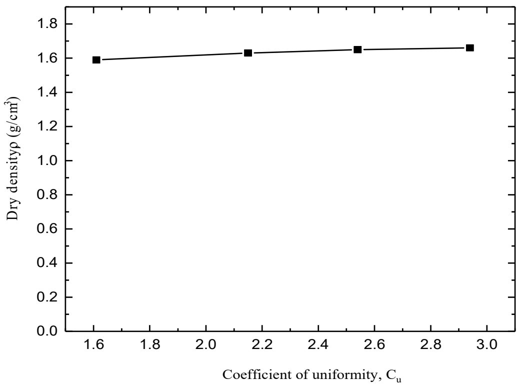


Figure 7 Density Vs Coefficient of uniformity

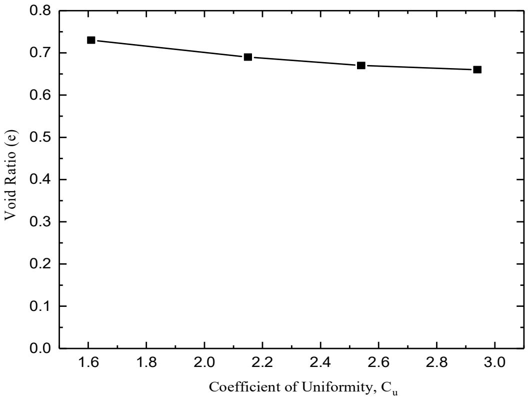
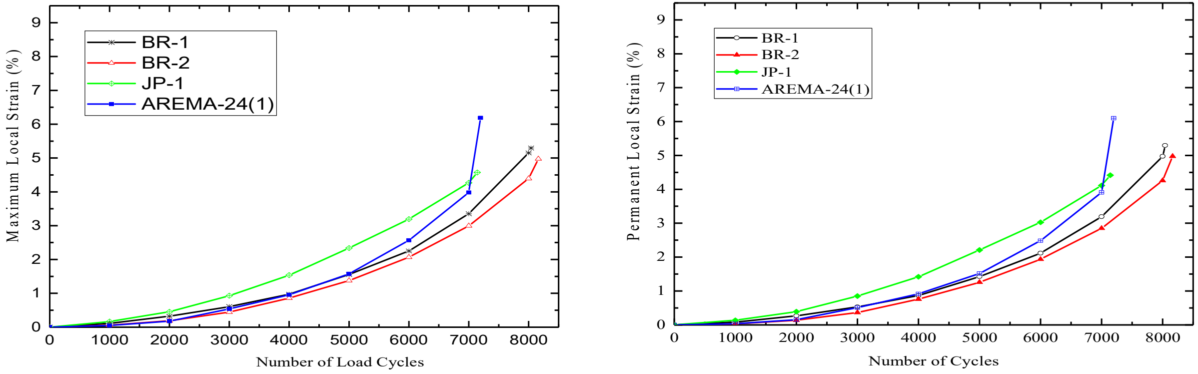


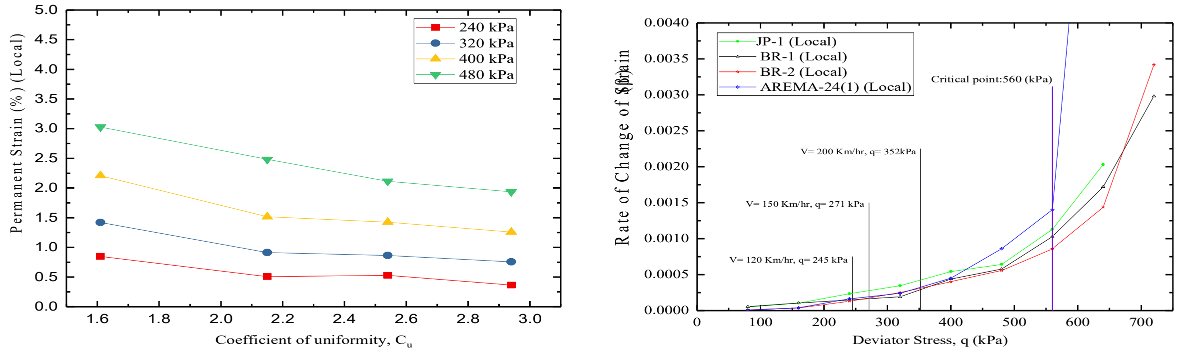
Figure 8 Void Ratio Vs Coefficient of uniformity

4.2 Maximum Strain and Permanent Strain

From the test results, it can be seen that both maximum and permanent strain in specimen decreases with increasing density of specimen (Figure 9). In general, axial strain at the same stress level decreases with increasing density. This is because the dense specimen is stronger due to higher internal friction. Hence, underwent less strain. Among the prepared specimens JP-1 had the lowest density and underwent maximum axial strain at the same stress level for the most part of the test but AREMA-24(1) underwent for maximum strain at the last cycle. Ballast stress in service for Bangladesh Railway is 245 kPa which was applied in between 3000 and 4000 cycles in the test. This result indicates that JP-1 will settle more than any other specimen in service life and it will be followed by AREMA-24(1), BR-1 & BR-2 respectively.



a) Maximum local strain load cycles b) Permanent local strain with load cycles



c) Permanent strain vs Cu d) Rate of change of strain Vs Deviator Stress

Figure 9 Strain relation of sample with deviator stress, load cycles and Co-efficient of uniformity, Cu

4.3 Rate of Increase of Strain (B)

The denser specimen had lower rate of increase of strain but there were very little differences among the values of β. Rate of increase of strain with deviator stress is quite low for all specimens up to 560 kPa deviator stress (Figure 6.9 to 6.12) and afterwards it increased rapidly. The design speed for broad gauge railway track in Bangladesh Railway is 120 Km/hr and associated deviator stress at that speed is 245 kPa. The values of β for all the specimens at 245 kPa are very low (i.e. less than 0.0005).

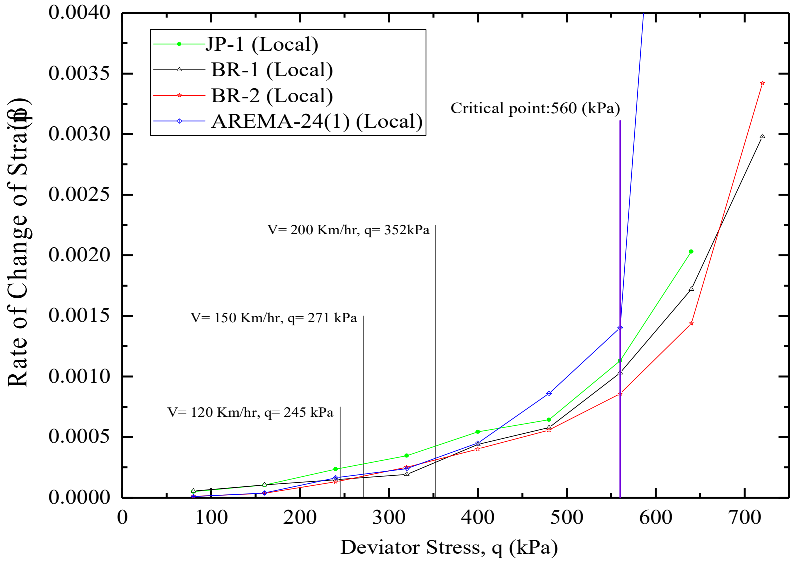


Figure 10 Rate of change of strain Vs Deviator Stress (Local)

4.4 Ballast Breakage Index and Ballast Grading

The larger-sized particles constitute the main skeleton of the ballast and the smaller particles reduce the contact forces between particles and minimize breakage. Friction among the larger sized particles provides the most strength to the ballast and also there are more sharp edges in larger sized particles. Hence, larger-sized particles break more than that of smaller-sized particles due to attrition and corner breakage which is quite obvious from the figure of the ballast breakage index (Figure 11). AREMA-24(1) has the highest ballast breakage index considering both cyclic and monotonic triaxial test and it is followed by JP-1, BR-2 and BR-1 respectively.

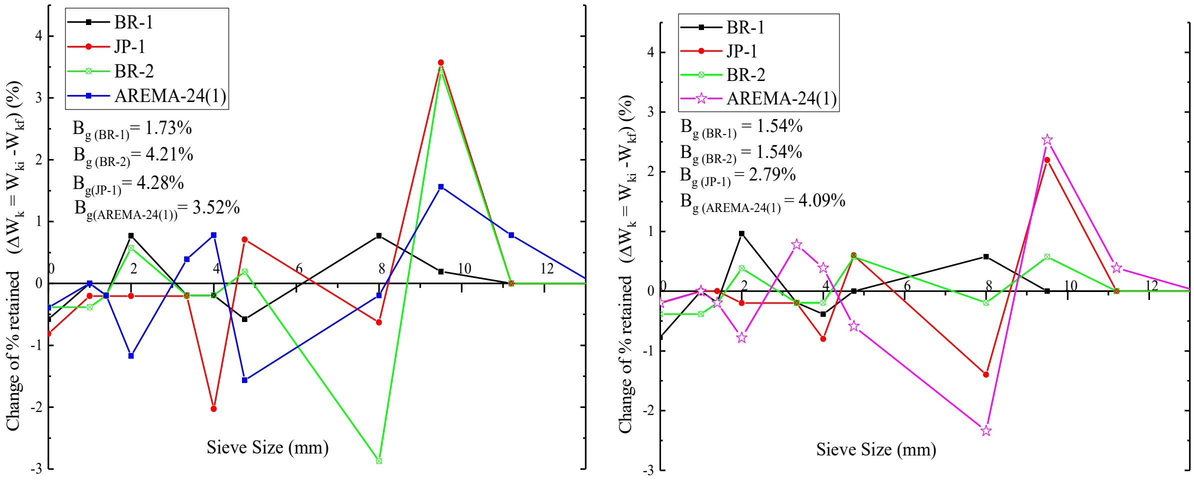


Figure 11 Ballast Breakage Index for BR-1,2, JP-1 and AREMA-24(1)

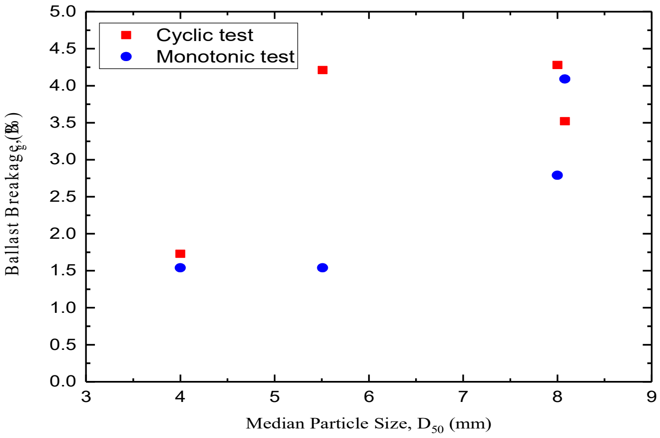


Figure 12 Ballast Breakage Vs D_{50}

4.5 Stress and Influence of Stress History

From the cyclic test results, it can be said that the ability to withstand maximum deviator stress increases with coefficient of uniformity (C_u) and density and it also increases the total amount of load the specimen can endure (Table 6, Figure 9(c) Figure 13 & Figure 14). It is because the internal friction among the particles increases with increasing density giving rise to the strength and durability of the ballast. However, the maximum deviator stress as per the monotonic test does not show any pattern (Figure 13). Diyaljee(1987) states that a previous stress history of more than 50% of the currently applied stress cyclic deviator stress, significantly decreases the plastic strain accumulation. It can be seen from the test results (Figure 4, Figure 5, Figure 6)that at the beginning strain at a certain stress level in cyclic loading is higher than that of monotonic loading but as the test progresses and the percentage (%) of previous stress history to currently applied stress increases, gradually difference between the cyclic strain and monotonic strain decreases. Finally, the monotonic strain becomes larger than the cyclic strain. It is due to the direct impact of previous stress history. Consequently, the maximum deviator stress as per cyclic triaxial tests for all specimens is more than that of monotonic triaxial tests.

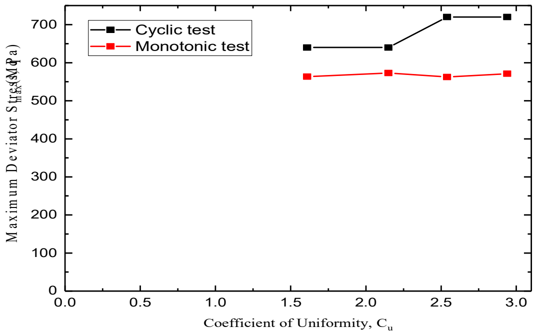


Figure 13 Maximum Deviator Stress Vs C_u

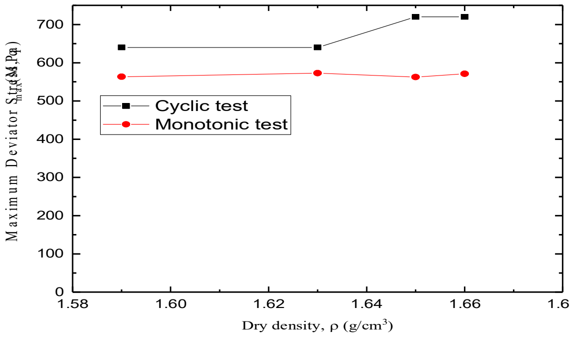


Figure 14 Maximum Deviator Stress Vs Dry density

5. PERFORMANCE EVALUATION OF THE TESTED GRADINGS

As per the cyclic triaxial test BR-1 and BR-2 withstand the same deviator stress (i.e. 720) and JP-1 and AREMA-24(1) endures 640 kPa deviator stress. BR-2 endures 8163 numbers of cyclic loading on the other hand BR-1, JP-1 and AREMA-24(1) endure 8040, 7141 and 7198 numbers of cyclic loading respectively. Whereas monotonic triaxial test results indicate, AREMA-24(1) is the strongest with 572.87 kPa deviator stress followed by BR-2, JP-1 and BR-1 with 571.05 kPa, 563.48 kPa and 562.55 kPa deviator stress respectively i.e. there is not much difference (Table 7). It is also observed that the sample stands more deviator stress before failure in case of cyclic loading than monotonic loading because more particle re-arrangement takes place in case of cyclic loading. Further, BR-2 underwent for lowest strain than that of others at the same level of stress as per cyclic and monotonic triaxial test and is followed by BR-1, AREMA-24(1) and JP-1 respectively (Table 7). Hence, it can be said that in the service life, BR-2 will settle less than others at the same loading. BR-1 possesses a significant percentage of fines (about 10%) and BR-2 possess about half the percentage of fines than BR-1. On the other hand, both JP-1 and AREMA-24(1) possess a very small percentage of fines even after the test. AREMA-24(1) has the highest ballast breakage index considering both cyclic and monotonic triaxial tests and it is followed by JP-1, BR-2 and BR-1 respectively.

Table 7 Comparison of test data for performance evaluation

Specimens	Cyclic Triaxial Test				Monotonic Triaxial Test		Ballast Breakage		Percentage of Fines (%)		
	Max. Deviator Stress (kPa)	Per. Strain* (%)	Max. Strain* (%)	Load Cycles endure	Max. Deviator Stress (kPa)	Local Strain (%)**	Cyclic	Mono-tonic	Before test	After Cyclic test	After Monotonic test
BR-1	720	3.194	3.353	8040	562.55	4.18	1.73	1.54	9.56	10.0	10.27
BR-2	720	2.850	2.995	8163	571.05	3.11	4.21	1.54	4.29	5.38	5.20
JP-1	640	4.114	4.267	7141	563.48	5.08	4.28	2.79	0	1.41	0.21
AREMA - 24(1)	640	3.910	3.982	7198	572.87	3.57	3.52	4.09	0	0.57	0.36

Note: * Strain is taken after the end of 7000 cycles i.e. after applying 0-560 kPa loading as this is the last full cycle loading common to all the specimens. ** Strain at 560 kPa.

6. CONCLUSION

For this study, two gradings (BR-1 & BR-2) are selected from the boundary of BRS, one grading is selected from Japanese Standard (JP-1), and another grading is selected from AREMA-24 (AREMA-24(1)). The experimental study was carried out on 1/5th scaled clean and dry ballast. A series of both monotonic and cyclic triaxial tests were conducted. On the analysis of both the physical properties and triaxial test results of the selected scaled ballast samples following conclusion is drawn:

i. The coefficient of uniformity of ballast grading has a significant impact on the density, strength, permanent, settlement, ballast breakage and durability of the specimens.

ii. Stress history has a significant impact on the reduction of strain in ballast.

iii. The result of the present study indicates that in dry and drained conditions the performance of BR-2 is the best followed by BR-1, AREMA-24(1) and JP-1 respectively. Hence, Bangladesh Railway Ballast Specification perform very well in dry and drained conditions. However, the void spaces among the particles of BR-1 and BR-2 are lower than the other two gradings.

REFERENCES

Selig, E. T., & Waters, J. M. (1994). *Track Geotechnology and Substructure Management*. Telford.

Jeffs, T.,& Tew, G. P. (1991). A review of track design procedures: sleepers and ballast, Vol. 2," Railw. Aust. BHP Res. Melb. Lab. Melbourne, Aust.

Thom, N. H. & Brown, S. F. (1988). The effect of grading and density on the mechanical properties of a crushed dolomitic limestone . 14th Australian Road Research Board (ARRB) Conference, Canberra, Austrailia, vol. 14, no.7.

Raymond, G. P. (1985). Research on railroad ballast specification and evaluation, no. 1006.

Marsal, R. J. (1967)."Large-scale testing of rockfill materials. J. Soil Mech. Found. Div., vol. 93, no. 2, pp. 27–43.

Raymond G. P. & Diyaljee, V. A.(1979).Railroad ballast sizing and grading. J. Geotech. Geoenvironmental Eng., vol. 105, no. ASCE 14556 Proceeding.

Roenfeldt, M. A. (1980). A study of mechanical degradation of a coarse aggregate subject to repeated loading.

Zakeri, J. A., Esmaeili, M., Kasraei, A.,& Bakhtiary, A. (2014). A numerical investigation on the lateral resistance of frictional sleepers in ballasted railway tracks. Proc Inst Mech Eng, Part F: J Rail Rapid Transit, 230 (2), pp. 440-449.

Klugar, K.(1978). A contribution to ballast mechanics. Railroad Track Mechanics and Technology, Elsevier, pp. 387–404.

Skoglund, K. A. (2002). A Study of Some Factors in Mechanistic Railway Track Design," Fakultet for ingeniørvitenskap og teknologi, Dr. ingeniøravhandling, 0809-103X; 2002:54, no. 3305, pp. 1–260.

Marschi, N. D., Chan, C. K. & Seed, H. B. (1972). Evaluation of properties of rockfill materials," J. Soil Mech. Found. Div., vol. 98, no. 1, pp. 95–114.

Dunn C. S. &. Bora, P. K. (1972). Shear strength of untreated road base aggregates measured by variable lateral pressure triaxial cell. J. Mater., vol. 7, no. 2.

Vallerga, B. A., Seed, H. B., Monismith, C. L. & R. S. Cooper, R. S. (1957). Effect of shape, size, and surface roughness of aggregate particles on the strength of granular materials," in Road and Paving Materials, ASTM International.

Holtz, W. G. & Gibbs, H. J. (1956). Triaxial shear tests on pervious gravelly soils," J. Soil Mech. Found. Div., vol. 82, no. 1, pp. 1–22.

Indraratna, B. & Salim, W. (2005). *Mechanics of Ballasted Rail Tracks: A Geotechnical Perspective*. Taylor & Francis, London.

Roner, C. J.(1985). Some effects of shape, gradation and size on the performance of railroad ballast. M. Sc. Degree Proj. report, Rep. No. AAR85-324P, Dep. Civ. Eng. Univ. Massachusetts, Amherst, Massachusetts.

Li, D., Hyslip, J., Sussmann, T. & Chrismer, S. (2002). *Railway Geotechnics*. CRC Press, pp. 92.

Diyaljee, V. A. (1987). Effects of stress history on ballast deformation. J. Geotech. Eng., vol. 113, no. 8, pp. 909–914.